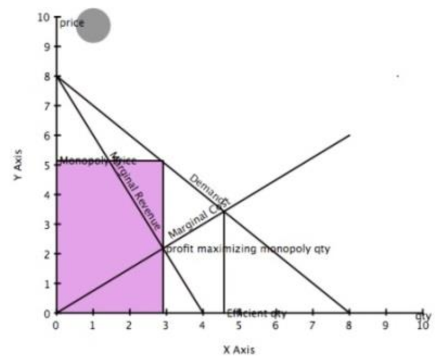
INTERCEPTS

You probably studied the chapter that taught you how to graph a straight line by plotting points. This chapter focuses on two special points that are easy to calculate and very important for future chapters math and for business, statistics, and science courses.



❑ CALCULATING INTERCEPTS

Consider the line $2x - 3y = 12$. We can find one point on the line very easily, by letting $x = 0$. This produces

$$2(\mathbf{0}) - 3y = 12$$

$$\Rightarrow \quad 0 - 3y = 12$$

$$\Rightarrow \quad -3y = 12$$

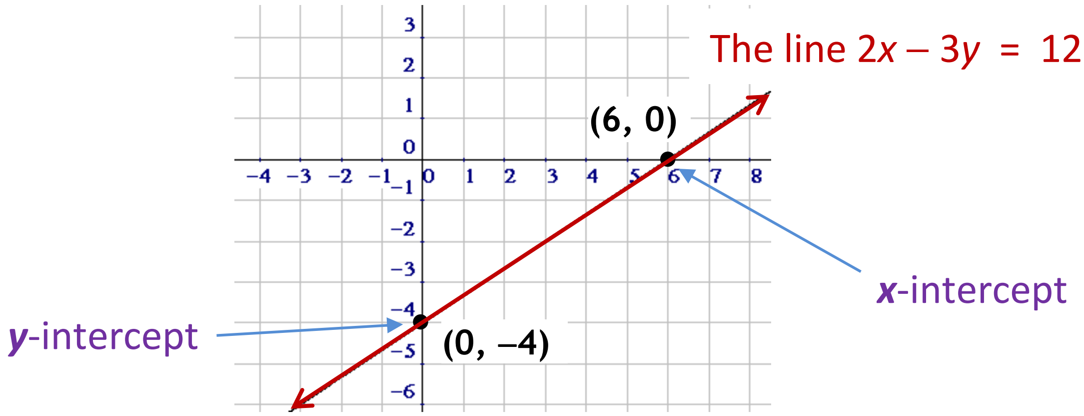
$$\Rightarrow \quad \underline{y = -4} \qquad \text{(divide each side of the equation by } -3\text{)}$$

Since we let $x = 0$, and got the result $y = -4$, this shows us that <u>the point **(0, –4)** is on the line</u>.

Now let's set y to 0. We obtain

$$2x - 3(\mathbf{0}) = 12 \;\Rightarrow\; 2x - 0 = 12 \;\Rightarrow\; 2x = 12 \;\Rightarrow\; \underline{x = 6}$$

We conclude that <u>the point **(6, 0)** is also on the line</u>. Since two points suffice to construct a line (although plotting more than two points is an excellent idea!), we'll graph our line right now using the points **(0, –4)** and **(6, 0)**:

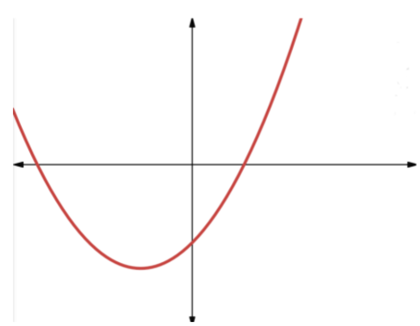


Notice that the point (6, 0), although certainly on the line $2x - 3y = 12$, is special because it lies on the x-axis. We call the point (6, 0) in this example the ***x-intercept*** of the line. Similarly, we call the point (0, –4) the ***y-intercept*** of the line.

Looking back at the calculations, we see that $x = 6$ (which gave us the x-intercept) was found by setting y to 0, and $y = -4$ (which yielded the y-intercept) was found by setting x to 0. Here's a summary of this easy way to find the intercepts of a line (and other graphs, too):

> To find ***x***-intercepts, set ***y*** = 0.
>
> To find ***y***-intercepts, set ***x*** = 0.

Of course, graphs such as circles and parabolas (which you might see later in this course) may have more than one x-intercept or more than one y-intercept (or even no intercepts of any kind). And when we get to horizontal and vertical lines pretty soon, you'll see that they may have only one kind of intercept, or they may even have an infinite number of intercepts!



This parabola has 3 intercepts

<u>EXAMPLE 1:</u> **Find the x-intercept and the y-intercept of the line $3x - 7y = 42$.**

<u>Solution:</u> To find the x-intercepts of this line (of any graph, in fact) we set $y = 0$ and solve for x:

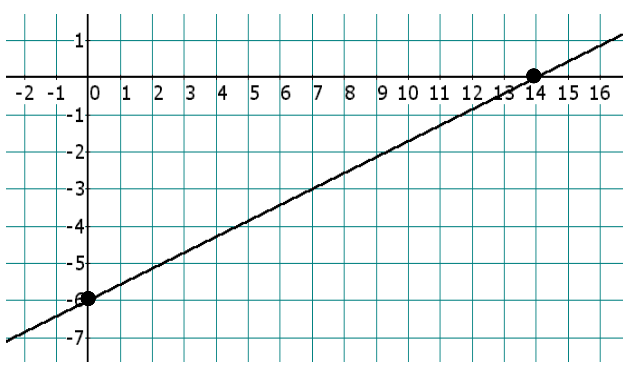
	$3x - 7y = 42$	(the line)
$\Rightarrow$	$3x - 7(\mathbf{0}) = 42$	(set $y = 0$)
$\Rightarrow$	$3x - 0 = 42$	(anything times 0 is 0)
$\Rightarrow$	$3x = 42$	(simplify)
$\Rightarrow$	$\dfrac{3x}{3} = \dfrac{42}{3}$	(divide each side by 3)
$\Rightarrow$	$x = 14$	

The x-intercept is therefore $\boxed{(14, 0)}$

Setting $x = 0$ to find the y-intercept gives

$$3(\mathbf{0}) - 7y = 42 \;\Rightarrow\; -7y = 42 \;\Rightarrow\; y = -6$$

and so the y-intercept is $\boxed{(0, -6)}$



The x-intercept is **(14, 0)**, the point where the line intersects the x-axis.

The y-intercept is **(0, –6)**, the point where the line intersects the y-axis.

<u>Note:</u> Some teachers will allow you to say that the x-intercept is 14, and some will require you to say that it's (14, 0), which is the notation promoted by this book. Ask your teacher what will be allowed.

Homework

1. Each of the following points (except one) is an intercept. Is it an x-intercept or a y-intercept?

   a. $(0, \pi)$ b. $(-99, 0)$ c. $(0, 0)$

   d. $(3\pi, 0)$ e. $(0, -3.7)$ f. $(7, 7)$

2. Find the x-intercept and the y-intercept of each line — be sure that every intercept you write consists of an ordered pair (that is, two coordinates, one of which must be 0):

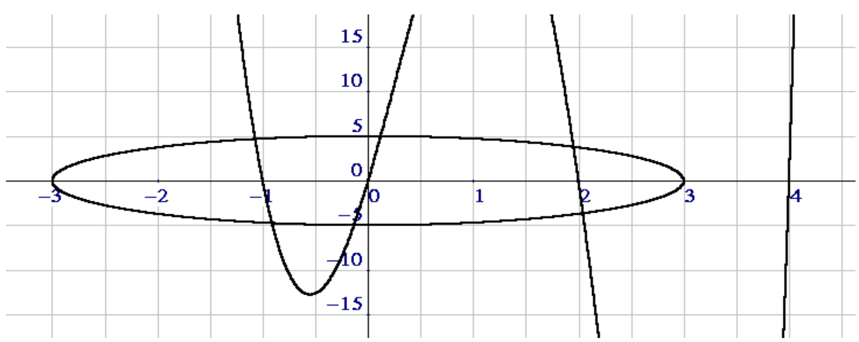
   a. $2x + y = 12$ b. $3x - 4y = 24$ c. $-4x + 7y = 28$

   d. $x - 7y = 7$ e. $y - 3x = 12$ f. $6x + 5y = 60$

   g. $4x - 3y = 2$ h. $8y + 3x = 1$ i. $3x - 9y = 0$

   j. $y = 7x - 3$ k. $y = -9x + 2$ l. $y = -x - 5$

3. Find <u>all</u> the intercepts of the following graph:



4. Sketch a graph that has NO intercepts of either kind.

Review Problems

5. Find all the intercepts of each line:

a. $y = 7x - 3$ b. $y = -9x + 8$ c. $y = x + 1$

d. $y = -x - 1$ e. $y = \frac{2}{3}x + 5$ f. $y = -\frac{1}{2}x - \frac{4}{5}$

g. $-2x - 7y = 0$ h. $4x + 8y = 6$ i. $18x - 17y = 2$

Solutions

1. a. y-intercept b. x-intercept c. both intercepts
d. x-intercept e. y-intercept f. neither

2. a. $(6, 0)$ $(0, 12)$ b. $(8, 0)$ $(0, -6)$ c. $(-7, 0)$ $(0, 4)$

d. $(7, 0)$ $(0, -1)$ e. $(-4, 0)$ $(0, 12)$ f. $(10, 0)$ $(0, 12)$

g. $(\frac{1}{2}, 0)$ $(0, -\frac{2}{3})$ h. $(\frac{1}{3}, 0)$ $(0, \frac{1}{8})$ i. $(0, 0)$

j. $(\frac{3}{7}, 0)$ $(0, -3)$ k. $(\frac{2}{9}, 0)$ $(0, 2)$ l. $(-5, 0)$ $(0, -5)$

3. $(-3, 0)$ $(-1, 0)$ $(0, 0)$ $(2, 0)$ $(3, 0)$ $(4, 0)$ $(0, -5)$ $(0, 5)$

4. A circle contained completely within Quadrant I would do the trick.

5. a. $(0, -3)$ $(\frac{3}{7}, 0)$ b. $(0, 8)$ $(\frac{8}{9}, 0)$ c. $(0, 1)$ $(-1, 0)$

d. $(0, -1)$ $(-1, 0)$ e. $(0, 5)$ $(-\frac{15}{2}, 0)$ f. $(0, -\frac{4}{5})$ $(-\frac{8}{5}, 0)$

g. $(0, 0)$ h. $(0, \frac{3}{4})$ $(\frac{3}{2}, 0)$ i. $(0, -\frac{2}{17})$ $(\frac{1}{9}, 0)$

❑ *To ∞ and Beyond*

A. Find all the intercepts of the graph of $x^2 + y^2 = 25$.

B. Find all the intercepts of the graph of $y = x^2 - 9$.

"Effort only fully releases its reward after a person refuses to quit."

– *Napoleon Hill*, American Author